



**School of Arts & Science**  
**MATHEMATICS DEPARTMENT**  
**MATH 262-X01**  
**Applied Differential Equations**  
**2008Q3**

## COURSE OUTLINE

### 1. Instructor Information

(a)	Instructor:	Raymond Lai		
(b)	Office Hours:	See the schedule below, or by appointment		
(c)	Location:	Centre for Business and Access (CBA) Room 152		
(d)	Phone:	370-4491	Alternative Phone:	
(e)	Email:	<a href="mailto:lai@camosun.bc.ca">lai@camosun.bc.ca</a>		
(f)	Website:	<a href="http://lai.disted.camosun.bc.ca/">http://lai.disted.camosun.bc.ca/</a>		

	Monday	Tuesday	Wednesday	Thursday	Friday
07:30-08:20	Office Hour	Office Hour	Office Hour	Office Hour	Office Hour
08:30-09:20		Office Hour		Office Hour	
09:30-10:20		Office Hour		Office Hour	
10:30-11:20	Office Hour	Office Hour	Office Hour	Office Hour	Office Hour
11:30-12:20	Office Hour	Office Hour		Office Hour	Office Hour
12:30-1:20	Office Hour	Office Hour		Math 262(X01) CBA 101	Office Hour
1:30-2:20	Math 262(X01) CBA 101	Math 262(X01) CBA 101		Math 262(X01) CBA 101	Math 262(X01) CBA 101
2:30-3:20	Math 262(X01) CBA 101	Math 262(X01) CBA 101			Math 262(X01) CBA 101

## 2. Intended Learning Outcomes

Upon completion of this course the student will be able to:

1. Classify a differential equation (DE) by type (ordinary differential equation ODE vs. partial differential equation PDE), order, and linearity.
2. Verify an implicit/explicit solution of an ODE.
3. Determine existence and uniqueness of solution of a first-order IVP.
4. Model real-life phenomenon with linear/non-linear DE (for example, vibration problems such as the spring-mass system, radioactive decay, mixture problems, draining a tank, falling bodies with/without air resistance).
5. Sketch approximate solution curves for a first-order IVP using direction field.
6. Solve various types of first-order DE: separable DE, linear DE (using integrating factor), exact DE and non-exact DE (by making it exact), homogeneous DE of a certain degree, Bernoulli DE.
7. Determine existence and uniqueness of solution of a  $n^{\text{th}}$ -order IVP.
8. Solve  $2^{\text{nd}}$ -order linear homogeneous/nonhomogeneous DE using the method of reduction of order.
9. Solve higher-order linear homogeneous/nonhomogeneous DE with constant coefficients.
10. Solve  $2^{\text{nd}}$ -order nonhomogeneous DE using the method of variation of parameters.
11. Solve Cauchy-Euler equation.
12. Solve systems of linear equations.
13. Classify a point for a DE as ordinary point, regular singular point, or irregular singular point.
14. Find power series solution of a DE about an ordinary point.
15. Find series solution of a DE about a regular singular point.
16. Apply the Frobenius Theorem to find series solution of a DE about a regular singular point.
17. Use a Laplace transform and its properties to solve an IVP of a linear DE with constant coefficients.
18. Use Euler and Runge-Kutta methods to find a solution of a DE.

## 3. Required Materials

(a)	Texts	Dennis G Zill, <i>A First Course in Differential Equations with Modeling Applications</i> , 8th Edition, Brooks/Cole, 2005.
(b)	Other	Graphing calculator (such as TI-89) that computes eigenvalues and eigenvectors is recommended

## 4. Course Content and Schedule

**Prerequisites** Math 260, Math 261

### *Organization*

In-class workload: 8 hours lecture per week

Out-of-class workload: 8 to 10 hours per week (or more for students with weak background)

31 March 2008 (Monday): First Lecture

14 April 2008 (Monday): Last day to drop quarter courses (Fee Deadline)

19 May 2008 (Monday): Victoria Day, College Closed

20 May 2008 (Tuesday): Last day to withdraw without a failing grade

2 June 2008 (Monday): Last day to request deposit refund after complete withdrawal ALL Quarter 4 '08 courses

13 June 2008 (Friday): Last Lecture

### *Tentative Course Content*

The course will follow the textbook fairly closely, covering the following topics:

#### ***Introduction to Differential Equations***

##### *Text (week) Topic*

- 1.1 (1) Definitions and Terminology
- 1.2 (1) Initial-Value Problems
- 1.3 (1) Differential Equations as Mathematical Models

#### ***First-Order Differential Equations***

##### *Text (week) Topic*

- 2.1 (1) Solution Curves Without a Solution
- 2.2 (2) Separable Variables
- 2.3 (2) Linear Equations
- 2.4 (2) Exact Equations
- 2.5 (2) Solutions by Substitutions
- Review (3) Classification of First-Order DE

#### ***Modeling with First-Order Differential Equations***

##### *Text (week) Topic*

- 3.1 (3) Linear Models
- 3.2 (3) Nonlinear Models
- 3.3 (3) Modeling with Systems of Differential Equations

## **Higher Order Differential Equations**

### *Text (week) Topic*

4.1 (4)	Linear Differential Equations: Basic Theory
4.2 (4)	Reduction of Order
4.3 (4)	Homogeneous Linear Equations with Constant Coefficients
4.4 (4)	Undetermined Coefficients – Superposition Approach
4.5 (4)	Undetermined Coefficients – Annihilator Approach
4.6 (5)	Variation of Parameters
4.7 (5)	Cauchy-Euler Equation
4.8 (5)	Solving Systems of Linear Equations by Elimination
4.9 (5)	Nonlinear Differential Equations

## **Modeling with Higher-Order Differential Equations**

### *Text (week) Topic*

5.1 (5)	Linear Models: Initial-Value Problems
5.2 (6)	Linear Models: Boundary-Value Problems
5.3 (6)	Nonlinear Models

## **Series Solutions of Linear Equations**

### *Text (week) Topic*

6.1 (6)	Solutions about Ordinary Points
6.2 (6)	Solutions about Singular Points

## **The Laplace Transform**

### *Text (week) Topic*

7.1 (7)	Definition of the Laplace Transform
7.2 (7)	Inverse Transform and Transforms of Derivatives
7.3 (7)	Operational Properties I
7.4 (8)	Operational Properties II
7.5 (8)	The Dirac Delta Function
7.6 (8)	Systems of Linear Differential Equations

## **Systems of Linear First-Order Differential Equations**

### *Text (week) Topic*

8.1 (8)	Preliminary Theory
8.2 (9)	Homogeneous Linear Systems
8.3 (9)	Nonhomogeneous Linear Systems
8.4 (9)	Matrix Exponential

## Numerical Solutions of Ordinary Differential Equations

### Text (week) Topic

2.6 (9)	A Numerical Solution
9.1 (9)	Euler Methods and Error Analysis
9.2 (10)	Runge-Kutta Methods
9.3 (10)	Multistep Methods
9.4 (10)	Higher-Order Equations and Systems
9.5 (10)	Second-Order Boundary-Value Problems

### Phase Plane Analysis (if time permits)

### Text (week) Topic

Notes (10)	Introduction to Phase Plane Analysis
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## 5. Basis of Student Assessment (Weighting)

### Assignment

- Refer to the end of this outline for a list of suggested exercises from the textbook. Solutions will be posted on the class's website. You can also find a copy of the "student resource and solutions manual" for the text (which contains complete solution for selected exercises) on reserve in the library.

### Term Tests

- There will be 5 term tests, one on each of the following weeks:

Week 2	week 4	week 6	week 8	week 10
7 – 11, April	21 – 25, April	5 – 9, May	19 – 23, May	2 – 6, June

- Complete understanding of the examples discussed in class and the suggested exercises from the textbook will be essential for success on the term tests.
- There is NO makeup. **Medical excuse must be accompanied by a physician's note.**
- Complete solutions will be posted online on the class's website.

### Final Examination

- The final exam will cover the entire course and will be 3 hours long.
- As stated on page 34 in the current college calendar 2007 – 2008, "students are expected to write tests and final examinations at the scheduled time and place." Exceptions will only be considered due to **emergency** circumstances as outlined in the calendar. Holidays or scheduled flights are not considered to be emergencies.
- Final examination period June 16 – 20 (specific date, time, and location TBA)

The final grade will be calculated according to the following breakdown:

5 Term Tests	Final Exam.
$5 \times 8\% = 40\%$	60%

Note: For grade inquiry, email your request with your Camosun student ID no.

## 6. Grading System

To pass the course, you need to obtain

- a minimum of 50% of the final exam, and
- an overall course mark of at least 60%.

### Standard Grading System (GPA)

Percentage	Grade	Description	Grade Point Equivalency
90-100	A+		9
85-89	A		8
80-84	A-		7
77-79	B+		6
73-76	B		5
70-72	B-		4
65-69	C+		3
60-64	C		2
50-59	D	Minimum level of achievement for which credit is granted; a course with a "D" grade cannot be used as a prerequisite.	1
0-49	F	Minimum level has not been achieved.	0

### Temporary Grades

Temporary grades are assigned for specific circumstances and will convert to a final grade according to the grading scheme being used in the course. See Grading Policy E-1.5 at [camosun.ca](http://camosun.ca) for information on conversion to final grades, and for additional information on student record and transcript notations.

Temporary Grade	Description
I	<i>Incomplete:</i> A temporary grade assigned when the requirements of a course have not yet been completed due to hardship or extenuating circumstances, such as illness or death in the family.
IP	<i>In progress:</i> A temporary grade assigned for courses that, due to design may require a further enrollment in the same course. No more than two IP grades will be assigned for the same course. <i>(For these courses a final grade will be assigned to either the 3<sup>rd</sup> course attempt or at the point of course completion.)</i>
CW	<i>Compulsory Withdrawal:</i> A temporary grade assigned by a Dean when an instructor, after documenting the prescriptive strategies applied and consulting with peers, deems that a student is unsafe to self or others and must be removed from the lab, practicum, worksite, or field placement.

## 7. Recommended Materials or Services to Assist Students to Succeed Throughout the Course

### LEARNING SUPPORT AND SERVICES FOR STUDENTS

There are a variety of services available for students to assist them throughout their learning. This information is available in the College calendar, at Student Services or the College web site at [camosun.ca](http://www.camosun.ca).

### STUDENT CONDUCT POLICY

There is a Student Conduct Policy **which includes plagiarism**. It is the student's responsibility to become familiar with the content of this policy. The policy is available in each School Administration Office, at Student Services and on the College web site in the Policy Section.

*How to do well in the course and where to get help*

1. Do not skip classes.
2. Start working on the exercises as soon as we finish a section.
3. Studying in groups is an efficient way to learn mathematics; however, make sure you can solve problems yourself.
4. Extra help available from assistant at the Interurban Math Room: Technologies Centre (TEC) Room 142 (phone: 370-4492). This drop-in centre is freely available for your use to work on math homework and to seek help from the tutor on staff (see hours posted on door).
5. Learning Support and Services for Students  
**There are a variety of services available for students to assist them throughout their learning. This information is available in the College Calendar, Registrar's Office or the College web site at <http://www.camosun.bc.ca>**
6. **Need a tutor/Want to become a tutor? Visit [http://www.camosun.bc.ca/resources/ses/tutors\\_list.php](http://www.camosun.bc.ca/resources/ses/tutors_list.php)**

Suggested examples to study and exercises to practice (Dennis G Zill, *A First Course in Differential Equations with Modeling Applications*, 8th Edition, Brooks/Cole, 2005)

Typos on answers at the end of the text:

#3.1.37(c) $33\frac{1}{3}$ seconds	#3.2.3 1,000,000; <b>52.9</b> months
#4.4.19 $y = c_1e^{-x} + c_2xe^{-x} - \frac{1}{2}\cos x - \frac{12}{25}\sin 2x - \frac{9}{25}\cos 2x$	
#7.3.75(a) $i(t) = \dots + \frac{10}{101}\cos\left(t - \frac{3\pi}{2}\right)U\left(t - \frac{3\pi}{2}\right) + \frac{1}{101}\sin\left(t - \frac{3\pi}{2}\right)U\left(t - \frac{3\pi}{2}\right)$	

Section (Page)	Examples	Exercises [Answers for even-numbered exercises]
1.1 (10)	1 to 5	1, 2, 3, 9, 15, 19, 20 $[y = x^2 \pm \sqrt{x^4 + 1}, \text{ both on } (-\infty, \infty)]$ , 21, 27, 28(a) $[m = 0, -1]$ , 29, 33, 34
1.2 (16)	1 to 5	1, 3, 9, 21, 22, 27, 31, 33 $[(b) y = \phi_1(x) = \sqrt{3(x^2 - 1)} (1 < x < \infty), y = \phi_2(x) = -\sqrt{3(x^2 - 1)} (1 < x < \infty), y = \phi_3(x) = \sqrt{3(x^2 - 1)} (-\infty < x < -1), y = \phi_4(x) = -\sqrt{3(x^2 - 1)} (-\infty < x < -1) (c) y = \phi_3(x)]$ , 39 $[y = 4e^{2x} + 3x^2 + 5]$ , 40 $[y = 2x^3 - x^2 - 5x - 8]$
1.3 (28)		13, 14 $\left[\frac{dh}{dt} = -\frac{5}{6h^{3/2}}\right]$ , 17, 18 $\left[\frac{d^2y}{dt^2} + \frac{15.6\pi s^2 g}{w}y = 0\right]$ , 21, 22, 23, 24 $\left[\frac{d^2r}{dt^2} = -\frac{kM}{R^3}r\right]$ , 28 $\left[\frac{dy}{dx} = -\frac{y}{\sqrt{s^2 - y^2}}\right]$ , 29, 36 $\left[(a) \frac{dr}{dt} = \frac{k}{\rho}, r = \frac{k}{\rho}t + r_0 (b) \frac{dv}{dt} + \frac{3k/\rho}{kt/\rho + r_0}v = g\right]$
2.1.1 (46)	1, 2	6, 9
2.1.2 (48)	3 to 5	25, 27, 39 (a) and (c), examples on handout (on suspended cable and suspension bridge)
2.2 (54)	1 to 4	9, 17, 19, 26 $[y = 2e^{-2t} + (1/2)]$ , 27, 39 (a) and (c)
2.3 (65)	2 to 7, remark (ii) on page 64	3, 6 $[y = x^2/2 - 1/2 + ce^{-x^2}, I = (-\infty, \infty), \text{ transient term } ce^{-x^2}]$ , 16 $[x = e^y - (2/y)e^y + (2/y^2)e^y + c/y^2, I = (-\infty, 0) \text{ or } (0, \infty), \text{ transient term } c/y^2]$ , 19, 30 $[y = \sin x \cos x - \cos x, -\pi/2 < x < \pi/2]$ , 32 $\left[y = \begin{cases} 1, & 0 \leq x \leq 1 \\ 2e^{1-x} - 1, & x > 1 \end{cases}\right]$ , 33, 36 $\left[y = e^{-e^x} \int_0^x e^{e^t} dt + e^{1-e^x}, y = e^{1-e^x}, y = 1\right]$
2.4 (73)	2 to 4	3, 13, 21, 27, 31, 33, 35, 36 $[x/y + x^2/2 + 5\ln y  - \cos y = C]$ , 39



2.5 (78)	1 to 3	2, 9, 15, 18 $[y^{-1} = -1 + (1/x) + (c/x)e^{-x}]$ , 21, 24 $[(x + y)^2 = 2x + c]$ , 27
2.6 (84)	1, 2	3
3.1 (98)	1 to 6	3, 4 [201], 9, 11, 12 [92%], 15, 21, 23, 25, 27, 29, 30 $[q = (1/1000) - (1/500)e^{-200t}$ , 0.0003 C, 0.1472 A, 0.001 C], 33, 35, 37
3.2 (108)	1, 2	1, 9, 10 [33.3 g, 150 g, 0 g and 0 g, 70 mins], 13, 14 [slower, in 38.16 mins], 15, 16 [823.843 ft], 17
3.3 (117)	example on page 114, example 1	6 $[x_1' = (1/50)x_2 - (3/50)x_1, x_2' = (3/50)x_1 - (7/100)x_2 + (1/100)x_3,$ $x_3' = (1/20)x_2 - (1/20)x_3]$ , 7, 12, 13
4.1.1 (137)	1 to 3	2, 3, 9, 10 $[-\pi/2 < x < \pi/2]$ , 11(a), 14(a) [No]
4.1.2 (138)	4 to 6, 8, 9	15, 17, 26 $[y = c_1e^{x/2} + c_2xe^{x/2}]$ , 27
4.1.3 (138)	10, 11	33, 35, 36 $[(a) y = 5 (b) y = -2x (c) y = 5 - 2x (d) y = (5/2) + 4x]$
4.2 (141)	1	Use reduction of order and not the formula for 3, 9, 10 $[y = x^{-3}]$ , and 15; 19, 20 $[y = e^{3x}, y = (x/3) + (4/9) + c_2e^{3x} + c_3e^{-x}]$
4.3 (147)	1 to 4	3, 6 $[y = c_1e^{5x} + c_2xe^{5x}]$ , 11, 25, 26 $[y = c_1e^{3x} + c_2e^{-3x} + c_3 \cos(\sqrt{2}x) + c_4 \sin(\sqrt{2}x)]$ , 27, 31, 32 $[y = (-7/4)e^{-x/2} + (11/4)e^{3x/2}]$ , 39
4.4 (158)	1 to 11	1, 2 $[y = c_1 \cos(3x/2) + c_2 \sin(3x/2) + 5/3]$ , 3, 4 $[y = c_1e^{-3x} + c_2e^{2x} - (x/3) - (1/18)]$ , 5, 15, 21, 25, 27, 29, 39
4.5 (166)	1 to 7	35, 39, 41, 43, 45, 47, 49, 51, 63, 69, 70 $[y = 8 - 6e^x + 3xe^x + 5x - (x^2e^x/2) + (x^3e^x/6)]$ , 71
4.6 (172)	1 to 3	3, 4 $[y = c_1 \cos x + c_2 \sin x + x \cos x - \sin x \ln \cos x ]$ , 15, 21, 23, 24 $[y = c_1 \cos(\ln x) + c_2 \sin(\ln x) + \cos(\ln x) \ln \cos(\ln x)  + (\ln x) \sin(\ln x)]$
4.7 (178)	1 to 6	21, 25, 27, 33
4.8 (182)	1 to 3	3, 9, 12 $[x = c_1e^{-t/2} + c_2e^{-t} + c_3e^t + 1, y = -3c_1e^{-t/2} - 2c_2e^{-t} - 2]$ , 15, 18 $[x = c_1e^{2t} + c_2 \cos t + c_3 \sin t + e^t, y = (3/2)c_1e^{2t} - c_2 \cos t - c_3 \sin t,$ $z = -2c_1e^{2t} - c_3 \cos t + c_2 \sin t]$ , 21
4.9 (188)	1 to 3	3, 5, 6 $[y = c_3e^{c_1x} - 1]$ , 7, 9 (b) and (c), 11, 13, 15
5.1.1 (207)	1, 2	3, 9, 11
5.1.2 (208)	3 to 5	21, 25, 27
5.1.3 (209)	6 to 8, section on pure resonance on page 204	30(a) $[x = e^{-t} \cos(2t) + 3 \sin(2t)]$ , 31, 33, 35, 37
5.1.4 (211)	9, 10	45, 47, 49, 53
5.2 (217)	1 to 3	1, 3, 9, 11, 15, 17, 27
5.3 (226)	1, 4	7, 15, 17, 18 $[(b) v = \sqrt{4x^2 - 36}, x = 3 \cosh(2t), v = 6 \sinh(2t) (c) 0.8184 \text{ secs}, 14.83 \text{ ft/s}]$

6.1.1 (248)	1	1, 3
6.1.2 (248)	2 to 6	15, 21, 23, 27, 29, 30 $[y = 2 - x - 2x^2 - (x^3/3) + (5x^4/12) + \dots]$ , 33, 34 $[y_1 = 1 + (x^2/2) - (x^3/6) + \dots, y_2 = x = (x^2/2) + (x^3/6) - (x^4/24) + \dots]$
6.2 (257)	1 to 5	5, 9, 21, 23, 27, 33
7.1 (283)	2 to 5	3, 4 $[(1 - 3e^{-s})/s + 2(1 - e^{-s})/s^2, s > 0]$ , 9, 12 $[e^{-5}/(s+2), s > -2]$ , 15, 16 $[(s-1)/(s^2 - 2s + 2), s > 1]$ , 21, 27, 28 $[(2/s^3) - 1/(s+9) + (5/s)]$ , 39
7.2.1 (292)	1 to 3	3, 5, 27
7.2.2 (292)	4, 5	31, 37, 38 $[y = (e^t/10) - (\sin(3t)/30) - (\cos(3t)/10)]$ , 39, 41
7.3.1 (301)	1 to 4	15, 16 $[2e^{-3t} \cos(5t) - (1/5)e^{-3t} \sin(5t)]$ , 23
7.3.2 (302)	5 to 9	41, 45, 55, 56 $[\mathcal{L}\{1 - U(t-4) + U(t-5)\} = (1/s) - (e^{-4s}/s) + (e^{-5s}/s)]$ , 57, 59, 65, 67, 69, 71, 79
7.4.1 (312)	1, 2	3, 9
7.4.2 (313)	3 to 6	21, 23, 25, 27, 28 $[s/(s^2 + 1)^2]$ , 32 $[e^t - t - 1]$ , 33, 37, 39, 45, 46 $[y = te^{-3t}]$ , 47, 48 $[i = 2 - 2e^{-100t} - 200te^{-100t} - 2U(t-1) + 2e^{-100(t-1)}U(t-1) +$ $+ 200(t-1)e^{-100(t-1)}U(t-1)]$
7.4.3 (314)	7, 8	50 $[1/(s(1 + e^{-as}))]$ , 51, 57, 58 $[x = 5 \sum (-1)^n (1 - e^{-(t-n\pi)} - (t-n\pi)e^{-(t-n\pi)})U(t-n\pi)]$
7.5 (318)	1	3, 7, 9, 13
7.6 (322)	1 to 3	9, 11, 15
8.1 (336)	1 to 7	3, 9, 13, 19, 21, 25
8.2.1 (351)	1, 2	9, 13
8.2.2 (351)	3 to 5	23, 25, 27, 29
8.2.3 (352)	6	33, 41, 45
8.3.1 (358)	1 to 3	3, 4 $[X(t) = c_1 \begin{bmatrix} -\sin(4t) \\ \cos(4t) \end{bmatrix} e^t + c_2 \begin{bmatrix} -\cos(4t) \\ -\sin(4t) \end{bmatrix} e^t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} 4/17 \\ 1/17 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{6t}]$ , 7, 9
8.3.2 (359)	4	11, 15, 21, 23, 27, 29, 31, 33
8.4 (362)	1	3, 4 $\begin{bmatrix} 1 & 0 & 0 \\ 3t & 1 & 0 \\ (3/2)t^2 + 5 & t & 1 \end{bmatrix}$ , 7, 9, 13, 15, 23
9.1 (372)	1, 2	9, 15, 17
9.2 (377)	1, 2	3, 17
9.3 (381)	1	3
9.4 (385)	1 to 3	1, 3, 5
9.5 (390)	1, 2	3, 8 $[y(1.000) = 0.0000, y(1.125) = -0.1988, y(1.250) = -0.4168,$ $y(1.375) = -0.6510, y(1.500) = -0.8992, y(1.625) = -1.1594,$ $y(1.750) = -1.4304, y(1.875) = -1.7109, y(2.000) = -2.0000]$ , 13