<u>Exponents</u> (Math 053 unit 4)

To prepare students for the MATH 053 unit 4 test, this supplement provides examples and exercises on exponents that are more complex than those found in text sections 10.1 and 10.2.

The table below includes examples of each of the definitions and rules for exponential expressions. Following the table are a number of exercises and answers, for students to practice using these rules on more complex expressions.

definitions and rules for exponents	general form	examples
Product rule: to multiply powers with like bases, use the same base and add the exponents.	$a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^3 = x^{2+3} = x^5$
Quotient rule: to divide powers with like bases, use the same base and subtract the exponent of the denominator from the exponent of the numerator.	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^5}{x^2} = x^{5-2} = x^3$
Power rule: to raise a power to a power, multiply the exponents.	$\left(a^m\right)^n = a^{mn}$	$\left(x^3\right)^4 = x^{3\cdot 4} = x^{12}$
Power rule for a product: to raise a product to a power, raise each factor to that power by multiplying the exponents.	$(ab)^n = \left(a^1b^1\right)^n = a^nb^n$	$\left(x^2y^5\right)^3 = x^{2\cdot 3}y^{5\cdot 3} = x^6y^{15}$
Power rule for a quotient: to raise a quotient to a power, raise both the numerator and the denominator to that power by multiplying the exponents.	$\left(\frac{a}{b}\right)^n = \left(\frac{a^1}{b^1}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{x^4}{y^3}\right)^2 = \frac{x^{4\cdot 2}}{y^{3\cdot 2}} = \frac{x^8}{y^6}$
Exponent of one: any number to the power of 1 is equal to that number, and any number with no exponent has an understood power of one	$a^1 = a$ $b = b^1$	$x^1 = x$ $y = y^1$
Exponent of zero: any nonzero number to the power of 0 is equal to 1.	$a^0 = 1$	$\frac{x^3}{x^3} = x^{3-3} = x^0 = 1$
Negative exponents: any nonzero number to a negative power is the reciprocal of that number to the corresponding positive power.	$a^{-n} = \frac{1}{a^n}$	$x^{-7} = \frac{1}{x^7}$
Special rules for negative exponents: due to the fact that a^n and a^{-n} are reciprocals.	$\frac{1}{a^{-n}} = a^n$ $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$	$\frac{1}{x^{-8}} = x^8$ $\frac{x^{-5}}{y^{-4}} = \frac{y^4}{x^5}$

The following examples apply the above rules to more complex exponential expressions and operations.

Product rule: to multiply exponential expressions: • multiply the numerical coefficients, then multiply the variables with like bases by adding the exponents • simplify using positive exponents	• $\left(-6x^3\right)\left(3x^{-5}\right) = (-6\cdot 3)\left(x^3\cdot x^{-5}\right) = -18x^{-2} = \frac{-18}{x^2}$ • $\left(7a^{-3}b^5\right)\left(-9a^7b^2\right) = (7\cdot -9)\left(a^{-3}\cdot a^7\right)\left(b^5\cdot b^2\right) = -63a^4b^7$ • $\left(x^3y^{-4}\right)\left(x^{-8}y^7\right) = \left(x^3\cdot x^{-8}\right)\left(y^{-4}\cdot y^7\right) = x^{-5}\cdot y^3 = \frac{y^3}{x^5}$
Quotient rule: to divide exponential expressions: • divide or simplify the numerical coefficients, then divide the variables with like bases by subtracting the exponents • simplify using positive exponents	• $\frac{36x^9}{-9x^5} = \left(\frac{36}{-9}\right)\left(\frac{x^9}{x^5}\right) = -4x^4$ • $\frac{32x^{-3}y^7}{-24x^6y^{-4}} = \frac{4}{-3}x^{-9}y^{11} = \frac{4y^{11}}{-3x^9}$ or $-1.\bar{3}\frac{y^{11}}{x^9}$ • $\frac{a^{-5}b^{-6}}{a^3b^{-2}} = \left(\frac{a^{-5}}{a^3}\right)\left(\frac{b^{-6}}{b^{-2}}\right) = a^{-8}b^{-4} = \frac{1}{a^8b^4}$

Power rule for a product: to raise a product to a power: • raise each factor to that power by multiplying the exponents • simplify using positive exponents	• $\left(2x^{-3}\right)^{-5} = \left(2^1x^{-3}\right)^{-5} = 2^{-5}x^{15} = \frac{x^{15}}{2^5}\ or\ \frac{x^{15}}{32}$ • $\left(-5a^{-4}b^5\right)^{-4} = (-5)^{-4}a^{16}b^{-20} = \frac{a^{16}}{(-5)^4b^{20}}\ or\ \frac{a^{16}}{625b^{20}}$ • $\left(-3x^{-7}y^{-3}\right)^3 = (-3)^3x^{-21}y^{-9} = \frac{(-3)^3}{x^{21}y^9}\ or\ \frac{-27}{x^{21}y^9}$
Power rule for a quotient: to raise a quotient to a power: • raise each factor of both the numerator and the denominator to that power by multiplying the exponents • simplify using positive exponents	• $\left(\frac{-2x^2y^{-3}}{3x^{-8}y^5}\right)^3 = \frac{(-2)^3x^6y^{-9}}{3^3x^{-24}y^{15}} = \frac{-8}{27}x^{30}y^{-24} = \frac{-8x^{30}}{27y^{24}}$ • $\left(\frac{-7x^4y^{-2}}{6x^{-1}y^{-3}}\right)^{-5} = \frac{(-7)^{-5}x^{-20}y^{10}}{6^{-5}x^5y^{15}} = \frac{6^5}{(-7)^5x^{25}y^5}$

Exercises Simplify using only positive exponents:

1. $\left(-4x^{-3}\right)\left(15x^{-9}\right)$ **2.** $\left(3k^{-2}\right)\left(-8k^7\right)$ **3.** $\left(x^{-5}y^2\right)\left(x^2y^{-4}\right)$ **4.** $\left(mn^{-2}\right)\left(m^8n^{-3}\right)$ **5.** $\left(-7a^{-3}b^3\right)\left(4a^{-1}b^{-5}\right)$

6. $\left(2x^2y^2\right)\left(3x^{-9}y\right)$ **7.** $\frac{27b^{-6}}{-9b^{-3}}$ **8.** $\frac{-10x^4}{-15x^{-2}}$ **9.** $\frac{x^{-1}y^3}{x^{-3}y^4}$ **10.** $\frac{a^3b^{-2}}{a^7b^4}$

11. $\frac{-28x^5y^6}{35x^{-2}y^8}$ **12.** $\frac{56a^{-5}b^{-4}}{-14a^9b^{-3}}$ **13.** $\left(-3x^4\right)^{-2}$ **14.** $\left(-5a^{-2}\right)^3$ **15.** $\left(4a^3b^{-4}\right)^2$

16. $\left(-6a^4b^{-5}\right)^{-2}$ **17.** $\left(3x^{-7}y^{-2}\right)^{-3}$ **18.** $\left(-4a^{-3}b^{-6}\right)^3$ **19.** $\left(\frac{-2x^2}{5x^{-2}}\right)^3$ **20.** $\left(\frac{3a^7}{4a^{-3}}\right)^{-2}$

21. $\left(\frac{2a^{-3}}{3a^4b^{-2}}\right)^5$ **22.** $\left(\frac{-5x^3y^{-1}}{4x^{-2}y^5}\right)^{-4}$ **23.** $\left(\frac{3x^2y}{-2x^7y^{-2}}\right)^3$ **24.** $\left(\frac{7a^{-3}b}{3a^{-5}b^3}\right)^{-2}$ **25.** $\left(\frac{-4x^{-1}y^{-4}}{5x^4y^{-2}}\right)^{-3}$

Answers

1. $\frac{-60}{x^{12}}$ **2.** $-24k^5$ **3.** $\frac{1}{x^3y^2}$ **4.** $\frac{m^9}{n^5}$ **5.** $\frac{-28}{a^4b^2}$ **6.** $\frac{6y^3}{x^7}$ **7.** $\frac{-3}{b^3}$ **8.** $\frac{2x^6}{3}$ **9.** $\frac{x^2}{y}$ **10.** $\frac{1}{a^4b^6}$

11. $\frac{-4x^7}{5y^2}$ **12.** $\frac{-4}{a^{14}b}$ **13.** $\frac{1}{9x^8}$ **14.** $\frac{-125}{a^6}$ **15.** $\frac{16a^6}{b^8}$ **16.** $\frac{b^{10}}{36a^8}$ **17.** $\frac{x^{21}y^6}{27}$ **18.** $\frac{-64}{a^9b^{18}}$

19. $\frac{-8x^{12}}{125}$ **20.** $\frac{16}{9a^{20}}$ **21.** $\frac{32b^{10}}{243a^{35}}$ **22.** $\frac{256y^{24}}{625x^{20}}$ **23.** $\frac{27y^9}{-8x^{15}}$ **24.** $\frac{9b^4}{49a^4}$ **25.** $\frac{125x^{15}y^6}{-64}$